Microworlds, Constructionism and Mathematics

Richard Noss
Celia Hoyles

Abstract: In this paper, we outline the notion of constructionism and how it might be put into practice through the design of “microworlds”, insulated and accessible islands of activity in which nuggets of relevant knowledge are encountered through specially designed tools, sequences of activities with suitably oriented pedagogies. In the second part of the paper, we describe the design, implementation and evaluation of a constructionist intervention, ScratchMaths, introduced in England where computing is compulsory throughout schooling (from 5 to 16 years). This case study highlights the tension between the fidelity of implementation of an innovation and its adaptation by teachers, especially the context of mathematics, which is high stakes for both teachers and learners.

Keywords: constructionism; microworlds; ScratchMaths; fidelity

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2 UCL Knowledge Lab, r.noss@ucl.ac.uk, ORCID: 0000-0001-6301-6489
3 UCL Knowledge Lab, c.hoyles@ucl.ac.uk, ORCID: 0000-0001-5724-7600
Resumen: En este artículo, esbozamos la idea del construccionismo y cómo puede ser puesta en práctica a través del diseño de “micromundos”, islas aisladas y accesibles de actividad, donde se encuentren pepitas de conocimiento relevante a través de herramientas y secuencias de actividades especialmente diseñadas y con pedagogías adecuadamente orientadas. En la segunda parte del artículo, describimos el diseño, implementación y evaluación de una intervención construccionista, ScratchMaths, introducida en Inglaterra, país en el que la computación es obligatoria para todos los niveles educativos (de los 5 a los 16 años). Este estudio de caso pone de manifiesto la tensión entre la fidelidad de una innovación al implementarla y su adaptación por parte de los profesores, especialmente en el contexto de las matemáticas, la cuál es una materia muy exigente tanto para los docentes como para los alumnos.

Palabras clave: construccionismo; micromundos; ScratchMaths; fidelidad

1. INTRODUCTION

We start with an observation of Seymour Papert who wrote on the dust-jacket of *Turtle Geometry* (Abelson and diSessa, 1981) that the book represented the “first of a new generation of mathematics textbooks”. In the event, it seems that this was too much to expect: even the most optimistic observer would have to admit that the changes did not quite play out as Papert envisioned. But note that there is, implicit in his observation that there would be something to write books about and people to read them, an implicit curriculum or at least part-curriculum.

How successful has this potential “new generation” of curricula and textbooks been in achieving its goal of fundamental change in what can be learned mathematically? A judgement is clearly a matter of perspective and for us change can be considered fundamental to the extent that it has been based on two key ideas: the idea of *constructionism*, and the idea of *microworld*. And like all big ideas, both of these ideas have been subject to pressure to dilute the fundamental in favour of the pragmatic. Understanding what this means and how far we have come in the last forty years will help us understand more why fundamental change is so difficult.
2. WHAT IS CONSTRUCTIONISM?

It is around 50 years ago that Seymour Papert launched the idea of constructionism. The central idea is that a powerful way for learners to build knowledge structures in their mind, is to build with external representations, to construct physical or virtual entities that can be reflected on, edited and shared. The widest interpretation, related continually in learned and popular journals, is that it is the same as constructivism: it is not.

Constructionism [...] shares constructivism’s connotation of learning as “building knowledge structures” irrespective of the circumstances of the learning. It then adds the idea that this happens especially felicitously in a context where the learner is consciously engaged in constructing a public entity, whether it’s a sand castle on the beach or a theory of the universe. (Papert, 1991, p1).

Constructionism therefore seeks, unlike constructivism, to inform a theory of pedagogy, by directly addressing the question of how best to help learners learn. By contrast, constructivism is a theory of how people learn, irrespective of the circumstances of that learning, or whether teaching is involved at all (for an introduction to constructivism, see for example, von Glasersfeld, 1989). As Papert goes on to put it, “the n-word”, constructionism rather than “the v-word”, constructivism, is aimed at trying to theorise strategies that align the way people learn with the ways it makes sense to help them learn, especially through the design of suitable artefacts. The word “especially” is crucial here, as it focuses attention on design: on the design of constructionist environments leading to the notion of a microworld, which we discuss later.

A classical example of a constructionist environment is one centred around Logo, the computer programming language derived from the artificial intelligence language, LISP (Harvey, 1997). Logo was, and still is in its various incarnations, a fully-fledged programming language by which people of all ages can program anything they can imagine – a picture, a robot, a videogame or a piece of music. Its latest instantiation in Scratch has millions of adults and children publishing and “remixing” what is essentially Logo/Lisp code with an intuitive visual and manipulable block-based interface.

The turtle (which was invented some years after the release of the first Logo) has many faces – primarily as a medium through which to engage even young children with ideas that are fundamental to mathematics (such as differential
geometry) and which would otherwise lie outside their reach. For example, some of the fundamental ideas of differential geometry, like curvature, are claimed to be more accessible through a medium of expression that capitalises on body syntonicity. The presence of a manipulable “concrete” object opens up three distinct but closely related affordances for the learner.

First, the constructionist environment seeks to afford a compelling medium in which to explore and learn from feedback (in different forms), much as one can master a foreign language by living in the appropriate country. Second, the learner can adopt a construction-based approach to learning designed so learners will encounter “powerful ideas” or intellectual nuggets. The notion of powerful ideas captures the notion of engagement with intellectual tools, ways of thinking that afford the learner access to key concepts and strategies in ways that connect to their own intuitive knowledge. (For a comprehensive review of the role of tools in the learning of mathematics, see Monaghan, Trouche & Borwein, 2016).

Third, constructionist tools seek to be expressive, that is they can be shaped by their users to construct new entities, in ways that emerge in activity. At the same time, tools constrain and shape what learners can do, think and learn and these constraints can be designed so as to be productive to learning. (See also the debate around the notions of situated abstraction, instrumental genesis and orchestration in Hoyles, Noss & Kent, 2004).

The affordances of Logo allow us to generalise the idea of constructionism beyond the case of Logo and its descendants. As Logo has evolved the ambient digital space around it has also evolved in tandem (Logo was invented some 30 years before the web!). In addition, the theory of constructionism has acquired more form and detail, inspiring designers to build more technologies that support its key objectives: Boxer, Scratch⁴, NetLogo⁵, ToonTalk⁶, and most recently hardware that finally is ubiquitous like the Raspberry Pi and the BBC Micro:bit. Numerous knowledge-focussed environments have now entered the constructionist arena, with similar visions for learning, such as the dynamic geometry systems in mathematics (Sinclair & Crespo, 2006) or Impromptu in music (Bamberger & Hernandez, 1999). Eisenberg (2003) has also added to this mix through his descriptions of environments that blend traditional and computational

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⁴ http://scratch.mit.edu/
⁵ http://ccl.northwestern.edu/netlogo/
⁶ http://www.toontalk.com/
material. Over the years, constructionism has also provided the framework for a fertile strand of research detailing trajectories of learning with the tools, which range widely over topics from topology to musical composition.

The discussion above illustrates that, as Papert was at pains to point out, constructionism seeks to develop knowledge structures in the mind alongside physical or virtual structures external to the mind, and as such is a theory of epistemology as much as pedagogy (see Harel & Papert, 1991). Papert explains that the distinction between instructionism and constructionism also concerns epistemology and not merely about two ways of thinking about the transmission of knowledge. Rather, the distinction “goes beyond the acquisition of knowledge to touch on the nature of knowledge and the nature of knowing” (Papert, 1993, p. 8). In other words, constructionism involves choosing or designing representations, engaging artefacts and suitably oriented pedagogies that together can bring about fundamental change in how to learn and what is learned.

A thought-provoking discussion of this epistemological shift has been explored by Wilensky and Papert who argue that constructionism has:

shifted the focus from the means to the object of learning... how the structure and properties of knowledge affect its learnability and the power that it affords to individuals and groups. (Wilensky & Papert, 2010, p. 1).

The name they give to this process is restructuration,

... the encoding of the knowledge in a domain as a function of the representational infrastructure used to express the knowledge. A change from one structuration of a domain to another resulting from such a change in representational infrastructure we call a restructuration. (ibid. pp. 2-3)

The example they give (Papert, 2006) is the shift (though not, of course, made for educational purposes) from Roman to Arabic numerals, a shift that made it possible for nearly everyone to calculate in ways that were hitherto obscure. Our challenge is to think beyond this example, and seek to identify where the computer presence has shifted not only how knowledge is spread and developed,

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7 This may be an appropriate point to direct the interested reader to Wilensky’s NetLogo, every copy of which contains not only the programming language itself, but dozens or hundreds of mathematical ideas that are ‘restructurations’ afforded by the new tool.
but the nature of knowledge itself, in scientific, social-scientific and humanities disciplines (see, for example, Resnick, 1995).

One of the persistent challenges to realising the constructionist vision, is the tension between aiming to teach specific content of, say, mathematics or music, and at the same affording the learner the experience of constructing, making, doing and problem solving by exploiting this content knowledge. These two aims are, of course, not antithetical, but neither is it obvious how to align them for pedagogical purposes. One solution that has evolved has been to design “microworlds”, insulated and accessible “islands” of activity in which nuggets of relevant knowledge are encountered in a “natural” way using digital tools.

3. MICROWORLDS

Hoyles (1993) describes the evolution of the microworld idea from its genesis in the artificial intelligence community, in which it was used to describe a relatively simple and constrained domain where computational systems could solve problems, to a more broadly conceived environment that served as a concrete embodiment of a knowledge domain or structure. The structure comprises tools that are extensible (so tools and objects can be combined to build new tools, but also transparent so their workings are visible, and rich in different representations. Edwards (1998) contrasts this “structural” view of a microworld with a “functional” view that prioritises its features as they become apparent in use, as learners explore, build and learn from feedback.

This functional view points to the importance of the way that knowledge actually grows in interaction with learners. As diSessa (2006) points out, traditional instruction fails to engage with how knowledge is actually built, piece by piece, and layer upon layer. There is a duality here: a successful microworld is both an epistemological and an emotional universe, a place where powerful (mathematical, or scientific, or artistic) ideas can be explored; but explored “in safety”, acting as an incubator both in the sense of fostering conceptual growth, and a place where it is safe to make mistakes and show ignorance. And, centrally these days, it is a place where ideas can be shared, remixed and improved. (For an earlier discussion of these twin aspects of engaging through building and sharing, see the idea of webbing as discussed in Noss & Hoyles, 2006).

The emotional component is more than incidental to the microworld idea: building and sharing implies learners care about what they are building and
also they approach learning as collaborative. As for the first point, Papert’s famous example in the preface to his book, Mindstorms, (Papert, 1980), tells a story that is not just about how much he learned about mathematics by playing with gears, but is about how he “fell in love” with gears, an intimate and consuming knowledge that he used as a model for future learning of mathematics. There are, of course, contexts other than mathematics and science that have been subject to the constructionist analysis: see discussion related to drawing and painting, in Clayson (2008) and Gargarian (1993).

But as well as an intellectual challenge for authentic engagement, there are issues that are fundamental to general goals of learning. In relation to mathematics, Confrey and her colleagues put it thus:

The importance of tapping into youth culture should not be underestimated in motivating and sustaining student educational progress. This is especially true for subjects like science and mathematics, which carry considerable social capital yet are easy for students to dismiss as irrelevant, boring and hard in a world of digital images, animations, easy information retrieval and communication. We need engaging environments, in which the mathematics is actually needed for students to achieve goals that they find compelling, and made visible to students and expressed in a language with which they can connect (Confrey et al., 2010, p. 20).

Authentic engagement of learners remains a major challenge for mathematics education today. It is not straightforward to design activities that encourage students to address the mathematical concepts at stake in ways that make it apparent that they have utility from the point of view of the learner. That is, although engagement with microworlds is intended to orient students towards mathematical ways of thinking through the structures put in place by the designer, learners must and will retain some autonomy. They need to take responsibility for their actions and the results they produce. This does mean, of course, that learning will never occur precisely as planned leading to an inevitable challenge: how to balance self-motivated activity while maximising the opportunity to encounter the planned powerful ideas (see the “Play Paradox”, Noss & Hoyles, 1996).

The terrain of mathematical learning is strewn with restructurings of mathematical knowledge: just one example is the set of “snapshots” in every issue of the International Journal of Computers in Mathematical Learning or in the “Models Library” that accompanies NetLogo. It is true there is no “new curriculum; indeed,
the very thought that there should be a “version” of mathematics that is designed for its learnability might be greeted by many mathematicians with dismay. But good examples do exist. One example, the study by Sacristán & Noss, 2008, shows how the formalisation of the idea of infinity in the form of program fragments can be an effective way to appreciate this deep and often inaccessible ideas.

We will consider in more depth below one coordinated attempt to build and evaluate a programming-based intervention, ScratchMaths (SM) comprising a set of microworlds designed for the exploration of key mathematical topics for children age 9-11 years.

4. THE SCRATCHMATHS PROJECT: A SEQUENCE OF MATHEMATICAL MICROWORLDS

We first set the context of the ScratchMaths project, which is important in framing its applicability and range. Since September 2014, all schools for students from 5-16 years in England are required to teach the mandatory National Computing Curriculum⁸ which includes designing and building programs. There are challenges in implementation with limited guidance on how to teach the proposed content, the specific levels of knowledge or understanding pupils should achieve at each stage of the curriculum and conceptual issues pupils are likely to encounter and how these should be addressed. Further challenges concern how to fit the new curriculum content into an already busy timetable, and crucially, from our perspective, how to forge cross-curricular links from computing to other curriculum areas.

The ScratchMaths (SM) project aimed to address some of these challenges by a longitudinal two-year intervention across English schools to promote 9-11 year-old students’ computational thinking in alignment with their mathematical thinking and reasoning. Thus SM designed a curriculum which addresses key aspects of the primary computing and mathematics curriculums for this age group. The intervention comprises six microworlds, called “modules”, three per year, and was designed by researchers working closely with four “design” schools to test and refine the student and teacher resources. (See https://www.ucl.ac.uk/ioe/research/projects/scratchmaths for freely available student and teacher resources.)

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We anticipated that the role of the teacher would be critical for successful implementation on the innovation and in what we designated as compulsory professional development sessions, considerable time was taken to share SM's overall pedagogic approach through the operationalisation of what we named the “5Es”: Explore, Explain, Exchange, Envisage and bridge. The aim was that these E's would together structure the whole classroom approach to the different activities in the SM curriculum (see Benton et al., 2017).

In the first year of SM curriculum for 9-10 year-old students, computational concepts were foregrounded with mathematical ideas more implicit in microworlds titled Tiling Patterns, Beetle Geometry and Collaborating Sprites. In the second year, the same students (now 10-11 years old) were introduced to mathematical concepts and mathematical reasoning explicitly through a programming approach along with a set of new computational concepts in microworlds titled: Building with Numbers, Exploring Mathematical Relationships, and Coordinates and Geometry.

Given the challenge of implementing a new curriculum, the SM teachers were provided with detailed guidance for navigation through the materials, which were themselves carefully structured and progressive. However, the SM team recognised the tension arising from their quest to provide comprehensive support and the need for teacher appropriation and autonomy, whereby teachers had space to customise the materials to suit their own goals and their students' needs. This is a variety of the play paradox referred to above and is now often referred to as the tension, or gap, between fidelity and adaptation. At the very least, the challenge is to reduce this gap and crucially, to avoid the emergence of "lethal mutations" (Brown & Campione, 1996), where the aims of the intervention are lost in its implementation.

The SM project was evaluated by an independent team of researchers whose focus was based on quantitative data collected by questionnaires and test outcomes, one being the outcome of a test of computational thinking (CT) administered at the end of the first year of the trial, and the second was the national mathematics Key Stage 2 test at age 11.10

The most significant first result was that children in ScratchMaths schools made greater progress in CT scores, compared to children in the control schools. Second, there was a greater increase in CT score for educationally disadvantaged students,

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9 For example, events, animation, control structures, variable, operators and expressions.
with educational disadvantage measured in the standard way in England by eligibility for free school meals as ascertained by family income. In short, disadvantaged students stood to gain most from SM, raising attainment in CT beyond that of the control group. And third, there were no differences between the boys and girls’ scores, a finding which is not insignificant when considered in the context of a widespread concern about girls’ of all ages preferences not to engage with programming.

The evaluators found however that there was no impact of SM on mathematical attainment as measured by the Key Stage 2 tests for 11-year olds at the end of Year 6. This is a disappointing result even though the measurement of success by these tests was always going to be problematic. An evaluation of impact on mathematics learning in relation to tests related to the mathematics addressed in the six modules might have produced different outcomes for example. We recognise that this discussion is necessarily speculative, and it is maybe seen as “making excuses” for lack of significant impact. Nonetheless, it does seem very clear from our teacher surveys that SM implementation was impeded, particularly in its second year, by two factors outside of the control of the innovation and more or less independent of it: high stakes testing in mathematics, and teachers teaching SM with little or no professional development. Related to the first point, the survey data showed that the fidelity of the implementation in the second year dropped dramatically as evidenced by curriculum coverage and time, and the limited engagement in professional development. Observations from school visits made it clear that SM time was negatively impacted by a focus on high-stakes testing in mathematics at the end of the school year, with more and more time given to revision and practice. In fact, the SM team learned from survey data that at least 25 schools had stopped teaching SM as early as January of the second year rather than continuing until the Key Stage 2 tests took place in May in order to give space for mathematics revision. In addition, because the professional development was measured at the school level, it is possible that in a high-fidelity school the switch to a teacher who had not taught SM in Year 5 is a case where an inconvenience becomes a lethal mutation. At worst, some teachers working on the Year 6 SM curriculum may not have participated in any SM training or received any school-based professional support whatsoever!
The SM project was ambitious in its scope, and perhaps over-ambitious in the range of topics that were re-worked within the microworld structure. But by far the most significant finding was the extreme susceptibility of the intervention to the initial conditions of teacher experience, expertise, and continuity. We believe that this finding generalises into learning environments based on the microworld idea.

The problem is that as the opportunities for collaborative learning, seamless and flexible interaction and access to information increase, there is no guarantee that these will enhance learning: in fact, the prospect of children working entirely alone without teachers might prove economically attractive even though from an educational point of view it is not. However much we would prefer otherwise, the SM study points to the undeniable fact that pedagogy and curriculum are indispensable and essential elements of microworlds – failure in this regard, for whatever reason, invites any intervention to morph into a lethal mutation.

This is only exacerbated by the momentum of technological change. As the opportunities for collaborative learning, seamless and flexible interaction and access to information increase, there is no guarantee that these will enhance learning. The focus here is on the creation of engaging culturally resonant artefacts, which simultaneously afford learners the opportunity to encounter powerful computational ideas.

We have emerged from an era of educational theory and practice in which the role of the teacher in microworld-design was subject for debate: now it is more essential than ever before. The mechanisms that underpin artefacts of all kinds are becoming less and less visible (no user serviceable parts), and it is inconceivable that students or their teachers will do justice to the potential of technology for mathematical learning without a change in the broader educational culture that comes to recognise the centrality of the teacher and her contexts in pursuing change.

REFERENCES


**RICHARD NOSS**

UCL Institute of Education, University College London
23-29 Emerald Street, London WC1N 3QS, UK