NOTAS Y NOTICIAS

What is Research in Mathematics Education, and What Are Its Results?*

Discussion Document for an ICMI Study

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As mathematics education has become better established as a domain of scientific research (if not as a scientific discipline), exactly what this research is and its results are have become less clear. The history of the past three International Congresses on Mathematical Education demonstrates the need for greater clarity. At the Budapest congress in 1988, in particular, there was a general feeling that mathematics educators from different parts of the world, countries, or even areas of the same country often talk past one another. There seems to be lack of consensus on what it means to be a mathematics educator. Mathematics education no longer means the same as *didactique des mathématiques* (if it ever did). Frech *didacticiens* refuse to translate their *didactique des mathématiques* into "mathematics education": a special English edition of the journal *Recherches en Didactique des Mathématiques* bears the title "Research in *Didactique* of Mathematics". *Die Methodik* (or the Polish *metodyka*, the Slovak *metodika*, and the like) have become obsolete. Does *research* mean the same as *recherche* or *investigación*? How do these words translate into other languages? Standards of scientific quality and the criteria for accepting a peper vary considerably among the more than 250 journals on mathematics education published throughout the world.

Despite this lack of consensus, publications appear that endeavor to depict the "state of the art" in mathematics education research. Individuals try to construct didactical theories. But reviewers never have trouble demonstrating the one-sidedness or incompletenss of such publications. Attempts to describe research in mathematics education or *didactique des mathématiques* or whatever other name is used may resemble the accounts of the legendary blind men exploring the legs of a huge elephant.

The ICMI study *What is research in mathematics education, and what are its results?* does not seek to describe the state of the art. Nor does it intend to tell anymore what research in mathematics education is or is not, or what is or

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is not a result. Instead, the organizers of the study propose to clarify the different meanings these ideas have for mathematics educators - to pinpoint the different perspectives, goals, research problems, and ways of approaching problems. The sudy will bring together representatives of the different groups of researches, allow them to confront one another's view and approaches, and seek a better mutual understanding of what we might be talking about when we speak of research in mathematics education.

Some Questions About Research

Such a wide-ranging discussion is badly needed in a community increasingly divided into specialized groups and cliques that are not always tolerant of each other. Besides mutual understanding within the community, however, there is also a need to explain the domain to representatives of other scientific communities, among which the community of mathematicians seens to be the most important. Nicolas Balacheff has observed:

> Most of us want to develop this research field within the academic community of mathematicians; this implies both the explanation of our purpose on a social ground (is there any need to develop such research?) and its relevance within the narrow academic world. For this reason, although it is not my sole concern, I have in mind the question of scientific standards, theses, publications, congresses, the employment of young academics in the field, and the connection between our research and research done in other fields.

Thus we need an "inner" identification of the research domain of mathematics education, as well as an outer vision from the perspectives domains.

One external domain, for exemple, is sociology. How is mathematics education organized and institutionalized? Where is research on mathematics education conducted? Where are theses on mathematic education defended? If a mathematics educator employed by a mathematics department has acquired his o her habilitation degree in, say, a department of pedagogy or philosophy (such a degree being unavailable at the employing institution), is he or she accepted as a full member of the community of mathematicians that awards doctoral or master's degree in mathematics? Are mathematics educators viewed as a part of the mathematics community? Similar questions arise when research in mathematics education is surveyed from other domains, including history, philosophy, anthropology, and psychology.

An approach from both within and outside the field of research in mathematics education raises the following questions, among others, to be discussed:

1. What is the specific object of study in mathematics education?

The object of study (*der Gegenstand*) in mathematics education might be, for example, the teaching of mathematics; the learning of mathematics; teaching/learning situations; didactical situations; the relations between teaching, learning, and mathematical knowledge; the reality of mathematics classes; societal views of mathematics and its teaching; or the system of education itself.

If a mathematics educator studies mathematics, is it the same object for him or her as it is for a mathematician who studies mathematics? What is mathematics as a subject matter? What is "elementary mathematics"? Analogous questions could be asked concerning the learner of mathematics as an object of study. Is it the same object for mathematics educator as it is for a psychologist or a pedagogue? Is the mathematics class or the process of learning in the school viewed in the same way by a mathematics educator and a sociologist, anthropologist, or ethnographer? Are questions of knowledge aquisition viewed the same way by a mathematics educator and an epistemologist?

The variety of activities offered at the ICMEs certainly distinguishes these congresses from, say, the international congresses of mathematicians. ICMI 7 was compared by some to a supermarket. Is there a unity in this variety? What gives unity to different kinds of study in mathematics education? Is this the object of research? Or is the object of research perhaps not even something held in common? Might the commonality lie in pragmatic aims of research in mathematics education?

2. What are the aims of research in mathematics education?

One might think of two kinds of aims: pragmatic aims and more fundamental scientific aims. Among the more pragmatic aims would be the improvement of teaching practice, as well as of students' understanding and performance. The chief scientific aim might be to develop mathematics education as a recognized academic field of research.

What might the structure of such a field be? Would it make sense to structure it along the lines of mathematical subject matter (e.g., the didactics of algebra or the didactics of geometry), of various theories or approaches to the teaching and learning of mathematics, or of specific topics or *problématiques* (research on classroom interaction and communication, research on students' understanding of a concept, etc.)?

Both kinds of aims seem to assume that it is possible to develop some kind of professional knowledge, whether that of a mathematics teacher, a mathematics educator, or a researcher in mathematical education. The question arises, however, whether such professional knowledge ca exist at all. Is it possible to provide a teacher, say, with a body of knowledge that would, so to say inevitably, ensure the succes of his or her teaching? In other words, is teaching an art or a profession (*un métier*)? Or is it perhaps a personal conquest? As Luigi Campedelli used to say, "*La didattica è, e rimane, una conquista personale*".

What does successful teaching depend on? Are there methods of teaching so sure, so objective, that they would work no matter who the teacher and students were? Are there methodos of teaching that are teacher-proof and methods of learning that are student-proof? If not, is there anything like objective fundamental knowledge for a researcher in mathematics education - something that any research could build upon, something accepted and agreed upon by all? Or will the mathematics educational community inevitably be divided by what is considered as belonging to this fundamental knowledge, by philosophies and ideologies of learning, by what is considered worth studying?

Many mature domains of scientific knowledge have become highly specialized into narrow subdomains. Is this the fate of mathematics education as well?

Or rather, in view of the interdisciplinary nature of mathematics education, must every researcher necessarily be a "humanist", knowing something of all domains and problems in mathematics education?

Although we aim at clarifying the notion of research in mathematics education as an academic activity, we should be careful not to fall into needlessly "academic" debates. After all, the ultimate goal of our research may be for a specific teacher in a specific classroom to be better equipped to guide his or her students as they seek to understand the world with the help of mathematics.

3. What are the specific research questions or *problématiques* of research in mathematics education?

Mathematics education lies at the crossroad of many well-established scientific domains such as mathematics, psychology, sociology, epistemology, cognitive science, semiotics, and economics, and it may be concerned with problems imported from these domains. But mathematics education certainly has its own specific *problématiques* that cannot be viewed as particular cases or applications of those from other domains. One question the ICMI study might address is that identifying and relating to each other the various *problématiques* specific to mathematics education.

There are certainly two distinct types of questions in mathematics education: those that directly or almost directly from the practice of teaching and those generated more by research. For example, the question of how to motivate students to learn a piece of mathematics (inventing interesting problems or didactical situations that generate a meaningful mathematical activity), or how to explain a piece of mathematics, belong to the first kind. The question of identifying students' difficulties in learning a specific piece of mathematics is also directly linked to practice. But questions of classifying difficulties, seeing how widespread a difficulty is, locating its sources, or constructing a theoretical framework to analyze it already belong among the research-generated questions. The problem is, however, that a difficulty may remain unnoticed or poorly understood without an effort to answer questions of the latter type; that is, without more fundamental research on students' understanding of a topic. Is it, therefore, possible to separate so-called practical problems from so-called research-generated problems?

Is it possible to admit the existence of two separate types of knowledge: the theoretical knowledge for the scientific community of researchers and the practical knowledge useful in applications for teachers and students? It might be helpful to reflect on the nature of these two types of knowledge, on relations between them, and on whether it would be possible to have a unified body of knowledge encompassing them both.

4. What are the results of research in mathematics education?

Any result is relative to a *problématique*, to the theoretical framework on which it is directly or indirectly based, and to the methodology through which it was obtained. This relativity of results, though commonplace in science, is often forgotten. One often interprets finding from biology, sociology, or mathematics education as if they were a kind of absolute truth. The reason may be that in these domains we really want to know the truth and not simply whether, if one proposition is true, some other proposition is also true. Questions of biology, sociology, or mathematics education can be of vital importance and fundamental to survival and well-being.

Two types of "findings" can be distinguished in mathematics education: those based on long-term observation and experience and those founded on specially mounted studies. Are the former less "scientific" than the latter? Geoffrey Howson offers an example:

> In the seventeenth century, Spinoza set out three levels of understanding of the rule of three (which, incidentally, can be viewed as an elaboration of the instrumental-relational model of Skemp and Mellin-Olsen expounded over three centuries later). This, like the well-known levels of the van Hieles, was based on observation and experience. On the other hand, for exemple, CSMS [Concepts in Secondary Mathematics and Science] used specially mounted classroom studies to develop and investigate similar hierarchies of understanding. Do we rule out the work of Spinoza as research in mathematics education? If we do, then we lose much valuable knowledge, especially that resulting from curriculum development. If we do not, then it becomes difficult to find a worktable definition [of research in mathematics education].

Balacheff points out that it may be difficult to contrast, in this way, the hierarchies obtained by the van Hieles and the CSMS group. Besides the different ways in which these hierarchies were obtained, the van Hieles and the CSMS group may not have been asking the same kind of question. "What are these questions?" asks Balacheff. "What is the validity of the answers they provide? How is it possible to relate them?"

Can a new formulation of an old problem be a research result? Can a problem be a result? Or a questioning of the theory related to a problem, a methodology, or a whole *problématique*? Can a concept be a result? It might be useful to have a definite categorization of the things we do in mathematics education, and of the things we thereby "produce".

Most people would probably agree that *making empirical investigations* is research. But is the *doing of practical things* research? Is *thinking* research? Can these activities be separated? Can a result be obtained without thinking and the doing of practical things? Should mathematics education be considered a science? Perhaps it is a vast domain of thought, research, and practice. What qualifies a domain of activity as scientific is the kind of validation and justification methods it uses. Proofs and experiments are considered scientific. But there are thoughts not validated in either of these ways that are valuable because they are filled with meaning.

What examples are there of what we consider results in mathematics education to be? What do we know today that we did not know before? What have learned about the processes of learning and teaching? What do we know about mathematics that mathematicians were not aware of before?

Can we identify some categories of results? One category might de *economizers* of thought. Any facts, laws, methods, procedures, or theories that are general enough to direc our experience and predict its results will give us increased power over our teaching and learning. Another category might be *demolishers of illusions*. Results that undermine our beliefs and assumptions are always valuable contributions to the field. A third category might be *energizers of practice*. Teachers welcome research that helps them understand what they teach and provides them with ideas for teaching. The development of teaching materials, activites, and challenging problems belongs to this category. Other categories of results might emerge from epistemological, methodological, historical, and philosophical studies.

5. What criteria should be used to evaluate the results of research in mathematics education?

How do we assess the validity of research findings? How assess their worth? Should we use the criterion of relevance? What about objective? Or originality? Should we consider the influence research has had on the practice of teaching? What other criteria should we use?

The first problem is to clarify the meaning of tems such as *truth, validity*, and *relevance* in the context of mathematics education. A related issue is the question of what is knowledge as such. This is an even more fundamental question than that of validation. If we knew what kind of knowledge mathematics education aims at, we would be better equipped for answering the question of methods of valididation.

It is also useful understand the ways in which research results are used. How have the results of research in mathematics education been applied? How do teachers use the research? How do policy makers use it? By clarifying the uses to which research is put, can we develop better criteria for assessing its validity?

Call for Papers

An ICMI Study on What is Research in Mathematics Education, and What are Its Results? will investigate the questions above, as well as others raised by various contributors, over the next year or so. The study will have two components: as invited *study conference* and a *publication* to appear in the ICMI Study series that will be based on contributions to and outcomes of the conference. The conference will be held in the spring of 1994 at a site to be determined (two possibilities are Japan and the USA), and the major outcomes of the study will be presented at the International Congress of Mathematicians in Zürich the following summer.

The International Program Committee (IPC) for the study invites readers to submit papers on specific problems or issues stimulated by this discussion document no later than **1 September 1993**. Contributors may wish to address either questions raised in the document or questions that arise in response to it.

Papers, as well as suggestions regarding the content of the study and the conference program, should be sent to *both* co-chairs of the IPC.

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(The above quotations from Nicolas Balacheff and Geoffrey Howson stem from the exchange of views during the preparation of this Discussion Document).